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A parameterised equation to estimate soil hydraulic conductivity in the field

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Abstract. The description of soil water dynamics using the Darcy–Buckingham approach involves the determination and use of soil hydraulic conductivity K *v.* soil water content θ functions. Many of the methods developed for the measurement of K are based on simplifying assumptions, such as the unit gradient and the choice of fixed models for the K(θ) relation. The need of quick, simple, and inexpensive methods to measure K(θ) in the field using a large number of replicates has also led soil physicists to develop simple methods. This paper presents a procedure that makes use of parameters of equations used to explain the internal water drainage process, and that naturally leads to the exponential character of the K(θ) relation. Results show that the parameterised equation represents a more rigorous estimation of K(θ), compared with the methods that assume unit gradient.

Additional keywords: unit gradient, internal drainage, soil water storage.

Introduction

The Darcy–Buckingham approach for the description of soil water dynamics involves the determination and use of soil hydraulic conductivity K *v.* soil water content θ functions. One of the most convenient field methods of the determination of K(θ) relations uses previously saturated soil profiles submitted to internal drainage, with a covered soil surface to prevent evaporation loss and rainfall gain. Some of the earliest studies on this subject are those of Youngs (1964), La Rue *et al*. (1968), and Davidson *et al*. (1969). Hillel *et al*. (1972) developed a procedure to handle data from internal drainage experiments, and this has been widely used to calculate soil hydraulic conductivity of field soils. The need for quick, simple, and inexpensive methods to measure K(θ) using a large number of replicates has also led soil physicists to develop simple methods, see for example Libardi *et al*. (1980), Sisson *et al*. (1980), and Chong *et al*. (1981), all of them using the unit gradient ($\partial H/\partial z = 1$) assumption, where H is the hydraulic potential head, taken as the sum of the matric potential head h and the gravitational potential head z. These methods were later compared by Jones and Wagenet (1984). Although very straightforward, the use of the resulting K(θ) relations remains restricted by their exponential character. Very small fluctuations in θ, even within the experimental error, yield large variations in K. Reichardt *et al*. (1998) made a critical analysis of the use of the K(θ) relations by the flux-gradient approach.

Most of these field methods are based on simplifying assumptions, such as the unit gradient and the choice of fixed models for the K(θ) relation. The procedure presented by Hillel *et al*. (1972) is an exception; however, their method of handling data needs to be updated in view of the computer tools now available. This paper presents a procedure, called parameterised since it makes use of parameters of models used to describe the internal water drainage process, which naturally leads to the exponential character of the K(θ) relation.

The use of parameters to define K(θ) relations has also been used in the laboratory, from 1-step outflow experiments (Kool *et al*. 1985) and from multi-step outflow experiments (van Dam *et al*. 1994) that use parameters of equilibrium relations, such as those of van Genuchten (1980) for the h(θ) relation and those of Mualem (1976) for the K(θ) relation. In this study, we use parameters that describe dynamic relations that prevail during the internal drainage process of a field soil, such as θ(t) and H(t).

Material and methods

In an internal drainage experiment, with the soil surface covered with an impermeable sheet, the integration of the Richards' equation leads to the expression used for the determination of the K(θ) relation at a chosen depth z = L:

$$K(\theta)_L = \frac{-\int_0^L \left[\frac{\partial \theta(z,t)}{\partial t}\right] dz}{\left[\frac{\partial H(z,t)}{\partial z}\right]_L} = \frac{-\frac{\partial S_L(t)}{\partial t}}{\left[\frac{\partial H(z,t)}{\partial z}\right]_L} \quad (1)$$

where z is the position coordinate, t the time, and S_L the soil water storage from soil surface down to depth L.

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To make K calculations using Eqn 1, a set of θ(z,t) and H(z,t) data was collected over a chosen internal drainage period t = 0 to t = T. Since the drainage process is decelerated, i.e. with rates ∂θ/∂t and ∂H/∂t decreasing asymptotically in time, the parameterised method suggests that the above datasets are tested to fit semi-logarithmic linear regressions of the types:

$$\theta z(t) = a + b \ln t \qquad (2)$$

$$S_L(t) = c + d \ln t \qquad (3)$$

$$H_L(t) = e + f \ln t \qquad (4)$$

It is important to realise that for t = 0, ln t = –∞, these models fail for very short times, and therefore, the regressions do not include θ_o, S_o, and H_o, the respective values corresponding to t = 0.

If the fitting of the experimental data according to Eqns 2–4 is significant (as already shown by Villagra *et al.* 1994), Eqn 1 can be 'parameterised' as follows:

$$\frac{\partial S_L(t)}{\partial t} = -\frac{d}{t} \qquad (5)$$

$$\left[\frac{\partial H(z,t)}{\partial z}\right]_L = \left[\frac{H(t)_{L+\Delta z} - H(t)_{L-\Delta z}}{2\Delta z}\right] = e' + f' \ln t \qquad (6)$$

where the hydraulic gradient G is calculated by finite differences, and $e' = (e_1 - e_2)/2\Delta z$ and $f' = (f_1 - f_2)/2\Delta z$; e_1, e_2, f_1, and f_2 are the coefficients of Eqn 4 for the regressions of H(t) at depths (L + Δz) and (L – Δz).

Substituting Eqns 5 and 6 into Eqn 1, we obtain a K *v.* t relation:

$$K(t)_L = \frac{-d/t}{e' + f' \ln t} \qquad (7)$$

Equation 7 allows the calculation of K for different times within the time interval used for the regressions. For each chosen time, there is a corresponding value of θ, given by Eqn 2; therefore, Eqn 7 can be transformed into a K *v.* θ relation for depth L. From Eqn 2 we have:

$$\ln t = (\theta - a)/b \qquad (8)$$

and

$$t = \exp[(\theta - a)/b] = \exp\left(\frac{-a}{b}\right).\exp\left(\frac{\theta}{b}\right) \qquad (9)$$

Introducing Eqns 8 and 9 into Eqn 7 yields:

$$K(\theta)_L = \left\{\frac{\left[-d.\exp\left(\frac{a}{b}\right)\right].\left[\exp\left(\frac{-\theta}{b}\right)\right]}{\left[e' + \frac{f'}{b}(\theta - a)\right]}\right\} \qquad (10)$$

To compare Eqn 10 with the most commonly used exponential K(θ) model:

$$K(\theta) = K_o \exp[\gamma(\theta - \theta_o)] \qquad (11)$$

in which K_o and θ_o are the values of K and θ for the saturated soil condition and γ is a regression constant, we have to use the transformation $\theta = (\theta - \theta_o)$, which takes Eqn 10 to the form:

$$K(\theta)_L = \left\{\frac{\left[-d.\exp\frac{1}{b}(a - \theta_o)\right].\exp\left[-\frac{1}{b}(\theta - \theta_o)\right]}{\left[e' + \frac{f'}{b}(\theta - a)\right]}\right\} \qquad (12)$$

which is the 'parameterised' K(θ) relation, including only parameters of regressions 2, 3, and 4. Eqn 10 is useful in the range in which the regressions 2, 3, and 4 are significant. It is important to recall that they fail for times very close to zero.

Comparing Eqns 11 and 12, it can be seen that γ = –1/b, that in Eqn 12 $K_o = -d.\exp[(a/b) + \gamma\theta_o]$, and that hydraulic gradient G = [e′ + (f′/b)(θ – a)] is present only in Eqn 12 and is a function of θ. G(θ) is the contribution of the gradient $[\partial H/\partial z]_L$ to the estimation of K(θ). G = 1 corresponds to the 'unit gradient' hypothesis (Reichardt 1993), which prevails when e′ = 1 and f′ = 0. Since Eqn 12 includes G(θ), the parameterised equation is more complete than the simple methods of Libardi *et al*. (1980) and Sisson *et al*. (1980). The estimate of K_o from the parameterised Eqn 12 can be shown to be exactly the same as that of Libardi *et al*. (1980) when their definition of a is equal to one.

To test Eqn 12, we used data of an internal drainage experiment carried out by Zevallos (1978) on a very homogeneous Oxisol profile (Red Yellow Latosol, sandy phase), of the county of Piracicaba, SP, Brazil, also used by Libardi *et al*. (1980) to test their method. After flooding experimental plots up to steady infiltration rates, the soil surface was covered with a plastic sheet in order to follow the internal drainage process, during a period of 45 days. During this period, a H(z,t) dataset was obtained through mercury manometer tensiometer readings, installed in triplicate at the depths of 0.15, 0.30, 0.45, 0.60, 0.75, 0.90, 1.05, 1.20, and 1.35 m from soil surface. The θ(z,t) dataset resulted from the conversion of tensiometer (h) data into θ data, using soil water characteristic curves made in the laboratory with undisturbed soil samples.

To use the parameterised Eqn 12, θ(z,t) and H(z,t) data have to be handled as shown below, presented in the form of a sequence of steps, as it was made by Hillel *et al*. (1972).

(1) Make θ (m^3/m^3) *v.* ln t (t = day) linear regressions, according to Eqn 2, for all depths. Judge their significance using a convenient statistical test. If significant, record parameters a and b for each depth (Table 1).
(2) Calculate soil water storage S_L (mm) using the trapezoidal method ($S_L = \bar{\theta}L$) for all depths z = L (mm) and times.
(3) Make S_L *v.* ln t linear regressions, according to Eqn 3, for all depths, if significant record parameters d (Table 1).
(4) Make H (m H_2O) *v.* lnt linear regressions, according to Eqn 4, for all depths, if significant record parameters e and f (Table 2).
(5) Calculate e′ and f′ according to Eqn 6 for z_i = L, using $\Delta z = z_{i+1} - z_i$ (m), e_1 and f_1 being parameters of regression of Eqn 4, for z_{i+1}; e_2 and f_2 for z_{i-1}.
(6) Write the parameterised Eqn 12 for any chosen depth L, introducing values of a, b, d, e′ and f′.

Results and discussion

Parameters a, b, c, d, e, and f are presented in Tables 1 and 2, together with the R^2 values of the respective coefficients of variation (CV). For θ *v.* lnt and S_L *v.* lnt regressions, their values are very high (>0.98) indicating that models of Eqns 2 and 3 describe very well the variation of these variables

Table 1. Regression parameters a, b, c, and d from Eqns 2 and 3 together with the R^2 coefficient values for the different depths

Depth (m)	a	b	R^2	c	d	R^2
0.15	0.2392	–0.0164	0.997**	35.878	–2.459	0.997**
0.30	0.3019	–0.0179	0.999**	81.157	–5.137	0.999**
0.45	0.2844	–0.0180	0.991**	123.813	–7.841	0.998**
0.60	0.3074	–0.0231	0.998**	169.926	–11.308	0.998**
0.75	0.2660	–0.0211	0.983**	209.830	–14.478	0.996**
0.90	0.2693	–0.0205	0.979**	250.218	–17.555	0.994**
1.05	0.2691	–0.0188	0.979**	290.578	–20.382	0.993**
1.20	0.2855	–0.0217	0.985**	333.408	–23.638	0.992**
1.35	0.2867	–0.0227	0.988**	376.419	–27.050	0.993**

**$P < 0.01$ (Owen 1962).

Table 2. Values of saturated soil water content θ_o, regression parameters e and f from Eqn 4 together with the R^2 coefficient values for the different depths

Parameters e′ and f′ were obtained from $(e_{i+1} - e_{i-1})/2(z_{i+1} - z_i)$ and $(f_{i+1} - f_{i-1})/2(z_{i+1} - z_i)$, respectively

Depth (m)	θ_o (m^3/m^3)	e	f	R^2	e′	f′
0.15	0.374	–107.287	–25.121	0.908**	–	–
0.30	0.389	–120.824	–25.272	0.927**	1.0423	–0.0216
0.45	0.390	–138.563	–24.472	0.949**	0.8817	–0.0776
0.60	0.419	–147.266	–22.945	0.952**	0.7296	–0.0536
0.75	0.386	–160.453	–22.863	0.968**	0.9243	0.00343
0.90	0.382	–174.997	–23.048	0.970**	0.9070	–0.0415
1.05	0.373	–187.662	–21.618	0.977**	0.8136	–0.0600
1.20	0.391	–199.411	–21.246	0.977**	0.7917	–0.0126
1.35	0.385	–211.410	–21.240	0.973**	–	–

**$P < 0.01$ (Owen 1962).

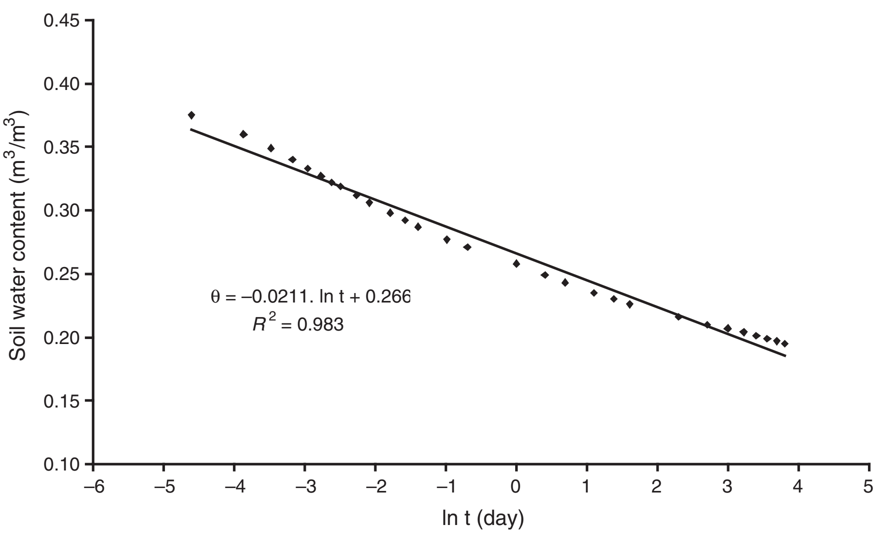

Fig. 1. Linear regression between soil water content θ and ln t (t in days) for the z = 0.75 m depth.

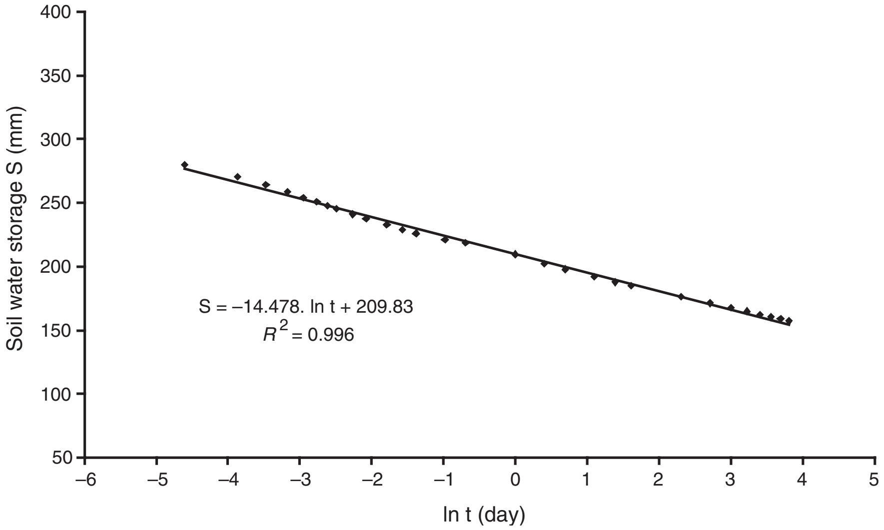

Fig. 2. Linear regression between soil water storage S and ln t (t in days) for the z = 0.75 m depth.

during the chosen time interval of the internal drainage process. As examples, Figs 1 and 2 show the experimental θ_{75} and S_{75} data for z = 0.75 m, and how well the solid lines (Eqns 2 and 3, respectively) fit the data.

Regressions of H *v.* ln t show somewhat smaller values of R^2 (Table 2) when considering the full range of data, but still high enough to assume that Eqn 4 describes field hydraulic water potential data. Figure 3 shows an example of H *v.* ln t plots, for the depths 0.60 and 0.90 m, on which it can be observed that for longer times, H data start deviating from the straight line behaviour, which should be a consequence of deviations from the unit gradient. As already stated, for the unit gradient to prevail, the parameters e′ and f′ should assume values equal to 1 and 0, respectively. This happens when in Eqn 6, $(e_1 - e_2)$ is equal to $2\Delta z$ and regression lines of Fig. 3 are parallel, i.e. $f_1 = f_2$, so that $f_1 - f_2 = 0$. Values of e′ and f′ shown in Table 2 differ somewhat from 1 and 0, which indicates that the unit gradient did not prevail during the internal drainage process, in agreement with statements of Reichardt (1993). The high values of R^2 presented in Table 2 indicate that parameterised equations of type 12 can be written for all depths. The missing values of e′ and f′ for the first and last depth are due to the finite difference procedure of Eqn 6. The parameterised equation for the depth L = 0.75 m, using the respective parameters found in Tables 1 and 2 for instance, is:

$$K(\theta)_{0.75} = \frac{4198.9 \exp\left[47.304\left(\theta - 0.386\right)\right]}{\left(0.9675 - 0.1626\,\theta\right)} \quad \text{(parameterised)} \quad (12a)$$

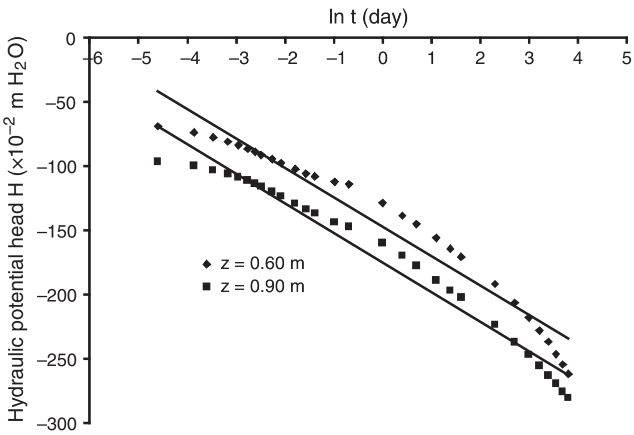

Fig. 3. Distribution of the hydraulic potential head H as a function of lnt (t in days) for the depths of z = 0.60 m and z = 0.90 m showing that, for longer times, H starts deviating from the straight line.

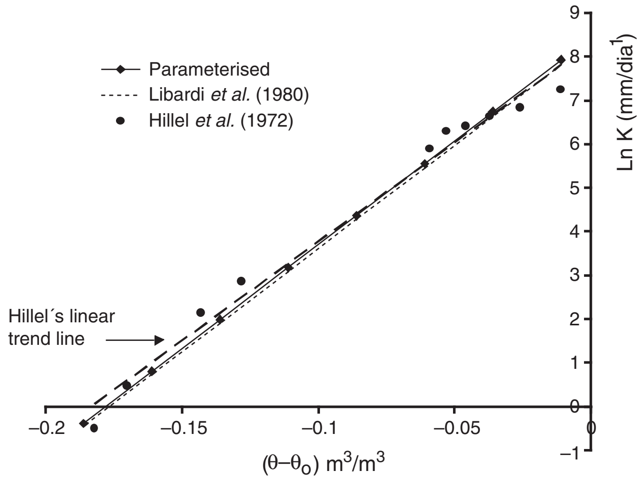

Fig. 4. Results showing that the new parameterised method (Eqn 12a) is comparable to the Hillel *et al.* (1972) and Libardi *et al.* (1980) methods, at z = 0.75 m.

For the same depth, the K(θ) relations calculated by the methods of Hillel *et al*. (1972) and Libardi *et al*. (1980) are, respectively:

$$K(\theta)_{0.75} = 4025.5 \exp[45.313\ (\theta - 0.386)] \quad \text{(Hillel)} \quad (13)$$

$$K(\theta)_{0.75} = 4198.9 \exp[47.304\ (\theta - 0.386)] \quad \text{(Libardi)} \quad (14)$$

Table 2 also presents the saturated field values of $\theta = \theta_o$, for t = 0, measured by tensiometers, which for the depth of 0.75 m is 0.386 m^3/m^3. Equations 12a, 13, and 14 are very similar, indicating that the new parameterised equation is comparable to the other two, well-established methods. Figure 4 also shows graphically how well the 3 methods mach. As already said, the coefficients K_o and γ are the same for Eqns 12a and 14, since both come from regressions of θ *v.* lnt, differing slightly from those of Eqn 13, which come from the final regression of K *v.* θ data. The scatter of the Hillel data comes from the finite differences Δθ/Δt and ΔH/Δz used in this method. The parameterised equation has the advantage over Libardi's of including the contribution of the gradient, which for this depth is G(θ) = (0.9675 – 0.1626*θ), differing from the unit gradient G = 1 built in to Eqn 14.

In conclusion, it can be said that the advantages of the new equation are: (*1*) over Hillel *et al*. (1972) by presenting an updated procedure to handle internal drainage data; and (*2*) over Libardi *et al*. (1980), Sisson *et al*. (1980), and Chong *et al*. (1981) by including quantitatively the effect of the hydraulic gradient on K calculations.

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